Oxford Cambridge and RSA

Tuesday 3 June 2025 – Afternoon

A Level Further Mathematics A

Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP

* 1 9 2 0 2 0 5 4 3 1 *

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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1 **In this question you must show detailed reasoning.**

Vectors **a** and **b** are given by $\mathbf{a} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$.

Determine, in either order

- **a.b**
- **a**×**b**. **[3]**

2 You are given that $-7-5\mathrm{i}$ is one root of the equation $x^3 + 10x^2 + 18x - 296 = 0$.

(a) Write down another complex root of the equation $x^3 + 10x^2 + 18x - 296 = 0$. **[1]**

(b) Using your answer to part **(a)**, express $x^3 + 10x^2 + 18x - 296$ as a product of a real linear factor and a real quadratic factor. **[3]**

3 Matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 1 & -3 \\ 4 & 8 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -3 & -3 \\ 1 & 2 \end{pmatrix}$.

(a) Find the matrix **AB**. **[1]**

(b) Verify that $\det(\mathbf{AB}) = \det(\mathbf{A}) \times \det(\mathbf{B})$. **[2]**

(c) Use matrices **A** and **B** to demonstrate that matrix multiplication is **not** commutative. **[2]**

The transformation represented by matrix **A** is denoted by T.

(d) Show that the point (2, −5) is **not** an invariant point under T. **[2]**

(e) Find the matrix which represents the inverse transformation of T. **[2]**

4 **In this question you must show detailed reasoning.**

Determine the sum of all cube numbers from 216 to 512 000 inclusive. **[4]**

5 **In this question you must show detailed reasoning.**

(a) Use an algebraic method to determine the two square roots of $-3+(4\sqrt{7})\mathrm{i}$.

Give your answers in the form $a+b\mathrm{i}$ where a and b are exact. **[5]**

(b) State the relationship between the two arguments of the two square roots found in part **(a)**. **[1]**

6 One of the regions bounded by two polar curves, C_1 and C_2, is used to model the face of a flat earring.

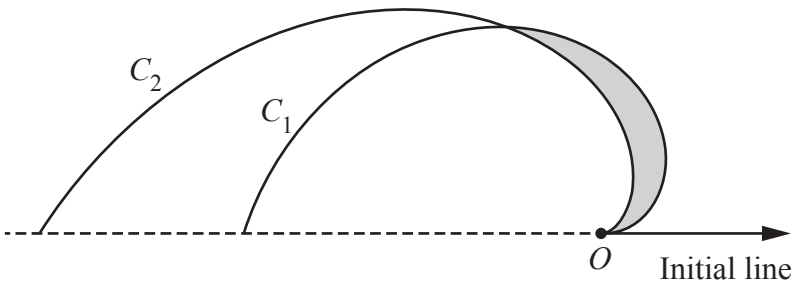
The polar equations of C_1 and C_2 are

$C_1: r = 2\theta$

$C_2: r = \theta^2$

where $0 \leqslant \theta \leqslant \pi$.

The curves C_1 and C_2 are shown in the diagram below with the region used to model the earring shaded.



You are given that C_1 and C_2 intersect at the pole O.

(a) Find the other point of intersection of C_1 and C_2. Give your answer in polar coordinates. **[2]**

(b) **In this question you must show detailed reasoning.**

Determine the area of the face of the earring. **[4]**

7 **In this question you must show detailed reasoning.**

(a) Express $\dfrac{-4x^2+5x-17}{x^3-x^2+3x-3}$ in partial fractions. **[6]**

(b) Hence determine the exact value of $\displaystyle\int_{\sqrt{3}}^{3} \frac{-4x^2+5x-17}{x^3-x^2+3x-3}\,\mathrm{d}x$. **[4]**

8 A function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = x\sinh 2x$.

(a) Prove by induction that $\dfrac{\mathrm{d}^{2n}\mathrm{f}}{\mathrm{d}x^{2n}} = 4^n(x\sinh 2x + n\cosh 2x)$ for $n \geqslant 0$ where $\dfrac{\mathrm{d}^0\mathrm{f}}{\mathrm{d}x^0}$ is defined as being equal to $\mathrm{f}(x)$. **[6]**

(b) Using the formula given in part **(a)**, determine the exact value of the coefficient of x^8 in the Maclaurin series for $x\sinh 2x$. **[3]**

(c) Use the Maclaurin series for e^x to verify your answer to part **(b)**. **[3]**

9 When light hits a certain photo-sensitive cell, at time $t = 0$, there is an electrical response in the cell which is denoted by $y(t)$ where both y and t are measured in suitable units. A student wishes to model this response, y.

In an attempt to model y, the student sets up the differential equation

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 10\,e^{-3t} \quad (*)$$

which is subject to the following conditions.

- $y = 0$ when $t = 5$
- $y \geqslant 0$ for all $t \geqslant 0$

(a) By substituting into a suitable differential equation, **verify** that $y = (A + Bt)e^{-3t}$ is a complementary function of the differential equation (*). **[2]**

(b) Determine the particular solution for y in terms of t. **[8]**

(c) Using your answer to part **(b)**, find the value of y immediately after the light hits the cell. **[1]**

The cell can be considered to be operating properly if $y < 4$ when $t = 1$.

(d) Discuss whether the cell can be inferred to be operating properly. **[2]**

10 **(a)** Find $\frac{1}{2}e^{-u}$ as a percentage of $\sinh u$ for the following values of u.

- $u = 2$
- $u = 5$ **[1]**

(b) Find, as a percentage of $\sinh u$, the difference between $\sinh u$ and $\cosh u$ for the following values of u.

- $u = 2$
- $u = 5$ **[1]**

A function f is defined for all integers n by $f(n) = \sinh(0.01n) - 5\cosh(0.005n) - 9\tanh n$.

(c) **In this question you must show detailed reasoning.**

With the help of suitable approximations, use an algebraic method to determine the smallest value of n for which $f(n) > 100$. You should **verify** your answer, once found, by direct calculation. You may assume that the required value of n is large. **[6]**

END OF QUESTION PAPER

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